Appendix B

Projection

B.1 Tangent or gnomonic projection

The gnomonic or tangent projection is a nonconformal map projection obtained by projecting points on the surface of sphere from the sphere's center to a plane that is tangent to a point in the surface of the sphere. All great circles in the sphere are projected as straight lines in a tangent projection (see Figures B.1 and B.2).

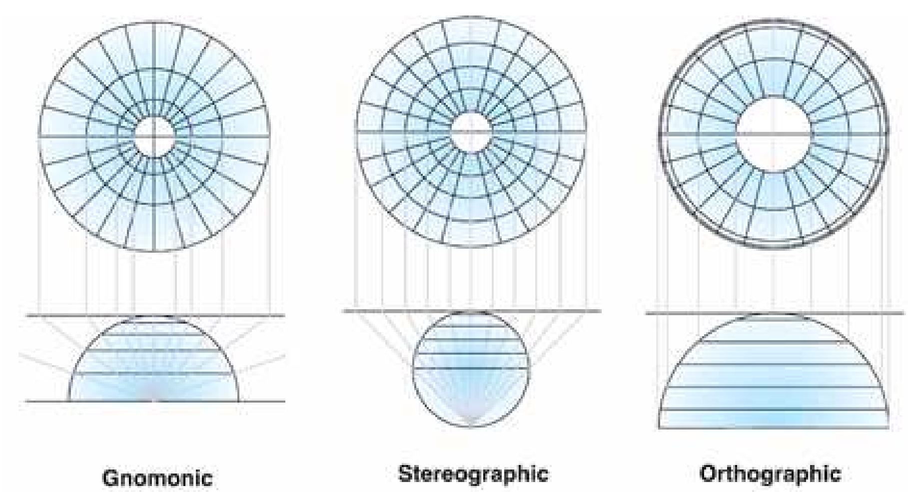


Figure B.1: Different planar projections.

B.2 DECAM image coordinates

DECAM images are given in FITS format under a modified tangent[1] projection called TPV projection. The TPV World Coordinate System is a tangent plane projection with a polynomial distortion function described by coefficients

[1]See Section B.1

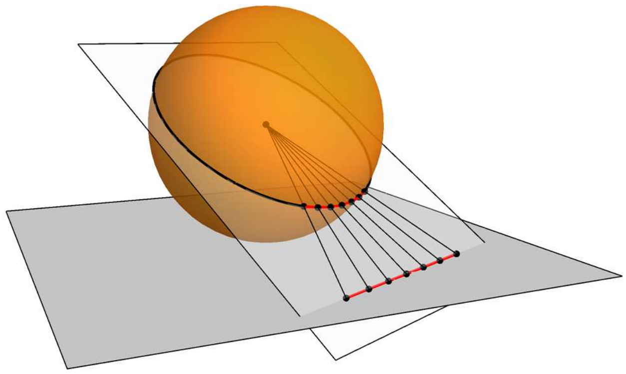


Figure B.2: Great circles always project as straight lines in the tangent or gnomonic projection.

provided in a set of PVi_m keywords. This transformation involves three steps from pixel coordinates into celestial coordinates in degrees:

1. The first step is the transformation between pixel coodinates $i,\ j$ into CCD coordinates $x,\ y$ in degrees. This step translates pixel coordinates into the observed image after the tangent projection, including the mirror imperfections.

$$x_{ij} = CD1\_1\ (i - CRPIX1) + CD1\_2\ (j - CRPIX2)$$
$$y_{ij} = CD2\_1\ (i - CRPIX1) + CD2\_2\ (j - CRPIX2),$$

where $i,\ j$ indices are in the range $1..N$ and $1..M$ if the image has $N \times M$ pixels, and the values CDi_j, $CRPIXi$ are given in the FITS file header. Note that the $CRPIXi$ refer to the tangent point of the transformation, which should be common to all CCDs in celestial coordinates. This means that the $CRPIXi$ values can be outside the image for multiple CCD instruments like DECAM.

2. The second step in the transformation is to convert the distorted CCD coordinates $x,\ y$ in degrees into undistorted tangent projection coordinates $\eta,\ \xi$ in degrees using the non–linear transformation given by the PV terms. This step corrects for the imperfections in the telescope mirrors:

$$\xi_{ij} = PV1\_0 + PV1\_1\ x_{ij} + PV1\_2\ y_{ij} + PV1\_3\ r_{ij} + PV1\_4\ x_{ij}^2 + PV1\_5\ x_{ij}\ y_{ij}$$
$$+ PV1\_6\ y_{ij}^2 + PV1\_7\ x_{ij}^3 + PV1\_8\ x_{ij}^2 * y_{ij} + PV1\_9\ x_{ij}\ y_{ij}^2 + PV1\_10\ y_{ij}^3$$
$$\eta_{ij} = PV2\_0 + PV2\_1\ x_{ij} + PV2\_2\ y_{ij} + PV2\_3\ r_{ij} + PV2\_4\ x_{ij}^2 + PV2\_5\ x_{ij}\ y_{ij}$$
$$+ PV2\_6\ y_{ij}^2 + PV2\_7\ x_{ij}^3 + PV2\_8\ x_{ij}^2 * y_{ij} + PV2\_9\ x_{ij}\ y_{ij}^2 + PV2\_10\ y_{ij}^3,$$

where $r_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}$ and where the PVi_j terms are given in the FITS file header.

3. The third step is to convert from tangent projection coordinates $\xi,\ \eta$ in degrees into celestial coordinates $RA,\ DEC$ in degress:

$$\alpha'_{ij} = \tan^{-1}\frac{\xi_{ij}/\cos(CRVAL2)}{1-\eta_{ij}\tan(CRVAL2)}$$
$$RA_{ij} = \alpha'_{ij} + CRVAL1$$
$$DEC_{ij} = \tan^{-1}\frac{\{\eta_{ij}+\tan(CRVAL2)\}\cos\alpha'_{ij}}{1-\eta_{ij}\tan(CRVAL2)},$$

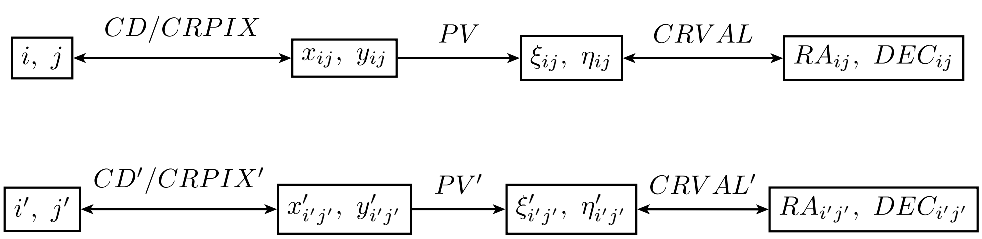
where $CRVALi$ are the coordinates of the tangent projection point and are given in the FITS file header. In principle, the $CRVALi$ values should be the same for all the CCDs in the DECAM field of view. They should also be similar between different images of the same field, but they are not because the telescope does not point with infinite precision.

B.3 Choosing a coordinate system

There are several alternatives to register (align) images depending on two factors:

- The reference system into which we would like to project the images
- The choice of projection grid in that particular space

Let's assume that we have two images with pixel coordinates $i,\ j$ and $i',\ j'$. Then the transformations required to bring the images to celestial coordinates are the following:



Thus, there are 7 different spaces where we can choose to project our images: $(i,\ j)$, $(x,\ y)$, $(\xi,\ \eta)$, $(RA,\ DEC)$, $(\xi',\ \eta')$ or $(i',\ j')$. Of them, only $(i,\ j)$ and $(i',\ j')$ will have regularly spaced grids in the input data. However, their overlap will not be regularly spaced, i.e. constant i' or j' lines plotted in the $(i,\ j)$ space will not be straight lines and their separation in $(i,\ j)$ space will not be constant for a constant difference of i' or j' and viceversa. This means that in practice it is always necessary to apply some non–linear image distortion to register images.

The $(x,\ y)$ space of any image does not appear to be favoured for any particular reason. It is only an intermediate steps between pixel coordinates and the tangent projection space or the celestial coordinates.

The tangent projection space $\xi,\ \eta$ of either image has the advantage of not significantly distorting objects close to the pole (unlike $RA,\ DEC$), but also that they correspond to perfect tangent projections, since the transformation between $\xi,\ \eta$ and $RA,\ DEC$ is that of a perfect tangent projection, removing